A Kinetics Of The Non-Equilibrium Universe. II. A Kinetics Of The Local Thermodynamical Equilibrium's Recovery.

Yu.G.Ignatyev, D.Yu.Ignatyev
Kazan State Pedagogical University,
Mezhlauk str., 1, Kazan 420021, Russia

Abstract

It has been researched the kinetics of the thermal equilibrium's establishment in an early Universe under the assumption of the recovery of interaction scaling of elementary particles in range of superhigh energies. The case of the thermal equilibrium's weak initial violation and basic cosmological consequences of the thermal equilibrium's violation have been researched.

1 Introduction

In the previous paper of one of the authors [1] it was shown, that in the case of the scaling behavior of the particles' cross-section of interaction in range of superhigh energies:

$$\sigma_{tot} \sim \frac{\mathrm{Const}}{s}, \tag{1}$$

where s is a kinematic invariant of the four-particle reaction (details see in [1]), the initial particles' distribution in the expanding Universe is not to be equilibrium, but can be random. In this paper we investigate the kinetics of the processes with elementary particles in the early Universe under the conditions of scaling of interactions with the purpose to clarify the boundaries of randomness of the initial particles' distribution. As the cross-section of elementary particles' interaction at that we will use an asymptotic cross-section of scattering, UACS, incorporated in papers [2], [3][1]:

$$\sigma_0(s) = \frac{2\pi}{s\left(1+\ln^2 \frac{s}{s_0}\right)} = \frac{2\pi}{s\Lambda(s)}, \tag{2}$$

where $s_0 = 4$ - the square of the total energy of two colliding Planck masses,

$$\Lambda(s) = 1 + \ln^2 \frac{s}{s_0} \approx \mathrm{Const}. \tag{3}$$

[1] As in the previous paper we will use a system of units $G = \hbar = c = 1$.

2 Kinetic equations for superthermal particles

2.1 A simplification of the relativistic integral of collisions

The relativistic kinetic equations for homogenous isotropic distributions $f_a(t,p)$ have the form (see the previous paper [1], details see in [4], [5]):

$$\frac{\partial f_a}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial f_a}{\partial p} = \frac{1}{\sqrt{m_a^2 + p^2}} \sum_{b,c,d} J_{ab \leftrightarrows cd}(t,p), \tag{4}$$

where $a(t)$ is a scale factor of the Friedmann's world:

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \rho^2(\chi)(\sin^2\theta d\varphi^2 + d\theta^2)]; \tag{5}$$

$$p^2 = -g_{\alpha\beta} p^\alpha p^\beta, \quad (\alpha, \beta = \overline{1..3}); \tag{6}$$

$J_{ab \leftrightarrows cd}(t,p)$ is an integral of four-particle reactions [6], [7]:

$$J_{ab}(t,p) = \pi^4 \int d\pi_b d\pi_c d\pi_d \delta^{(4)}(P_a + P_b - P_c - P_d) \times$$

$$\times [(1 \pm f_a)(1 \pm f_b)|f_c f_d \overline{M_{cd \to ab}|^2} - \tag{7}$$

$$-(1 \pm f_c)(1 \pm f_d) f_a f_b |\overline{M_{ab \to cd}|^2}],$$

characters $\pm$ correspond to bosons $(+)$ and fermions $(-)$, $M_{i \to f}$ are invariant amplitudes of scattering (line means the average by particles' states of polarization), $d\pi_a$ is a normed differential of volume of the a particle's momentum space:

$$d\pi_a = \sqrt{-g} \frac{\rho_a dp^1 dp^2 dp^3}{(2\pi)^3 p_4}, \tag{8}$$

ρ_a is a factor of degeneration.

Let us simplify an integral of four-particle interactions (7), using the properties of the distributions' isotropy $f_a(t,p)$. For the fulfilment of two inner integrations we will proceed to the local c.m. system, integration in which is carried out by the elementary way. After the inverse Lorentz transformation and conversion to the spherical system of coordinates in the momentum space we obtain ([8]):

$$J_{ab}(p) = -\frac{2S_b + 1}{8(2\pi)^4 p} \int\limits_0^\infty \frac{qdq}{\sqrt{m_b^2 + q^2}} \times$$

$$\times \int\limits_{s_-}^{s_+} \frac{sds}{s + m_a^2 - m_b^2} \frac{1}{16\pi\lambda^2} \int\limits_{-\lambda^2/s}^{0} dt \overline{|M(s,t)|^2} \int\limits_0^{2\pi} d\varphi \times$$

$$\{f_a(p_4) f_b(q_4)[1 \pm f_c(p_4 - \Delta)][1 \pm f_d(q_4 + \Delta)] -$$

$$-f_c(p_4 - \Delta) f_d(q_4 + \Delta)[1 \pm f_a(p_4)][1 \pm f_b(q_4)]\},$$

where

$$\Delta = -\frac{ts}{\lambda^2}\left[p_4 - q_4 - (p_4+q_4)\frac{m_a^2-m_b^2}{s}\right] -$$

$$-\cos\varphi\sqrt{-\frac{ts}{\lambda^2}\left(1+\frac{ts}{\lambda^2}\right)}\times$$

$$\times\left[4p_4q_4\left(1-\frac{m_a^2+m_b^2}{s}\right)-\frac{\lambda^2+4m_b^2p_4^2+4m_a^2q_4^2}{s}\right]^{1/2},$$

$$\lambda^2 = s^2 - 2s(m_a^2+m_b^2)+(m_a^2-m_b^2)^2, -$$

a function of a triangle, s, t are the kinematic invariants (see [1]),

$$s_\pm = m_a^2 + m_b^2 + 2(p^4q^4 \pm pq).$$

It is necessary at that to bear in mind the definition of the total cross-section of an interaction [9], (see also the previous paper [1]):

$$\sigma_{tot} = \frac{1}{16\pi\lambda^2}\int\limits_{-\lambda^2/s}^{0} dtF(s,t), \tag{9}$$

where we denoted as is generally accepted:

$$F(t,s) = \overline{|M(s,t)|^2}. \tag{10}$$

In the ultrarelativistic limit:

$$\frac{p_i}{m_i}\to\infty \Rightarrow \frac{s}{m_i^2}\to\infty; \quad \lambda\to s^2, \tag{11}$$

aforecited expressions are greatly simplified:

$$J_{ab}(p) = -\frac{(2S_b+1}{32(2\pi)^4p}\int\limits_0^\infty dq\int\limits_0^{4pq}\frac{ds}{s}\int\limits_0^1 dxF(x,s)\int\limits_0^{2\pi}d\varphi\times$$

$$\{f_a(p)f_b(q)[1\pm f_c(p-\Delta)][1\pm f_d(q+\Delta)]-$$

$$-f_c(p-\Delta)f_d(q+\Delta)[1\pm f_a(p)][1\pm f_b(q)]\}, \tag{12}$$

where a new variable was incorporated (see [1]):

$$x = -\frac{t}{s} \tag{13}$$

and

$$\Delta = x(p-q) - \cos\varphi\sqrt{x(1-x)(4pq-s)}. \tag{14}$$

2.2 An integral of collisions for a distributions' weak deviation from the equilibrium

Let us investigate at first a weak violation of the thermodynamical equilibrium in the hot model, when the main part of particles, $n_0(t)$, lies in the thermal equilibrium state, and only for the minor part of particles, $n_1(t)$ the thermal equilibrium is violated (see Fig. 1):

$$n_1(t) \ll n_0(t). \tag{15}$$

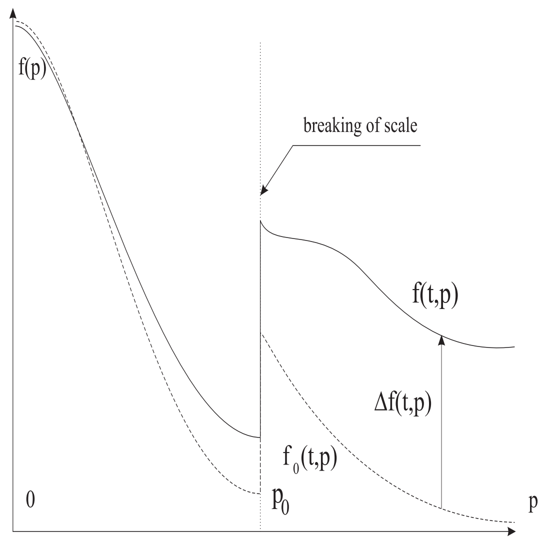

Figure 1: *The schematic representation of the distribution function's deviation from the equilibrium.*

Henceforth in this paper we will suppose that distribution functions different not greatly from the equilibrium ones in the range of small values of energy, smaller than certain unitary limit, $p = p_0$ (or $T = T_0$), below which scaling is absent, and can be violated at energies, above the limit:

$$f_a(p) \approx \begin{cases} f_a^0 = \dfrac{1}{\exp(\dfrac{-\mu_a + E_a(p)}{T}) \pm 1}, & p < p_0; \\ \\ \Delta f_a(p);\ f_a^0(p) \ll \Delta f_a(p) \ll 1, & p > p_0, \end{cases} \tag{16}$$

where $\mu_a(t)$ are the chemical potentials, $T(t)$ is a temperature of the equilibrium component of plasma. Thus, in range $p > p_0$ it can be observed the anomaly great number of particles in comparison with the equilibrium one, but minor at that (. (15)) in comparison with the total number of equilibrium particles.

Let us investigate the process of relaxation of the distribution $f_a(p)$ to the equilibrium $f_a^0(p)$. The problem in such a setting for the special case of the

initial distribution $f(t=0,p)$ was solved earlier in [2], [3]. Here we will give the general solution of this problem. At that, as it will be obvious from the further arguments, the cosmological plasma can be formally considered as a two-component system- equilibrium with the distribution $f_a^0(t,p)$, and nonequilibrium *superthermal*, with the distribution $\delta f_a(t,p) = \Psi(t,p)$, where number of particles in the nonequilibrium component is small, but energy density, as a matter of fact, is random.

Let us investigate an integral of collisions (12) in range

$$p \geqslant p_0 \gg T. \tag{17}$$

In consequence of the inequality (16) in this range we can neglect collisions of superthermal particles between themselves, not going beyond the account of the superthermal particles' scattering on equilibrium particles. Therefore the value of one of momentums in the integral of collisions, $P' = p - \Delta$, or $q' = q + \Delta$ must lie in a thermal range, another's value - in a superthermal one, outside the unitary limit. A subintegral value of the integral of collisions is extremely small outside this range. In consequence of this circumstance we can neglect the second member in curly brackets (12), since it can compete with the first in asymptotically small variation ranges of variables x and φ: $x(1-x) \lesssim T/p \to 0$. The statistical factors of type $[1 \pm f_a(p')]$ in the first member of an integral (12) can noticeably differ from one besides only in the range of momentums' thermal values. As a result the integral of collisions (12) in the investigated range of momentums' values can be written down in a form:

$$J_{ab\leftrightarrow cd}(p)|_{p\geqslant p_0} = \frac{(2S_b+1)\Delta f_a(p)}{(2\pi)^3 p}\times$$

$$\times \int\limits_0^\infty \frac{q f_b^0(q)dq}{\sqrt{m_b^2+q^2}} \int\limits_{2p(q^4-q)}^{2p(q^4+q)} \frac{ds}{16\pi} \int\limits_0^1 dx F(x,s). \tag{18}$$

Using the definition of the total cross-section of scattering (12), we obtain from (18):

$$J_{ab\leftrightarrow cd}(p)|_{p\geqslant p_0} =$$

$$= \frac{(2S_b+1)\Delta f_a(p)}{(2\pi)^3 p} \int\limits_0^\infty \frac{q f_b^0(q)dq}{\sqrt{m_b^2+q^2}} \int\limits_{2p(q^4-q)}^{2p(q^4+q)} \sigma_{tot} s(s) ds. \tag{19}$$

Substituting eventually an expression for σ_{tot} in the inner integral in form of UACS, (2), carrying out an integration with a logarithmic accuracy and summing up the obtained expression by all channels of reactions, we find finally:

$$J_a(p)|_{p\geqslant p_0} =$$

$$= -\Delta f_a(p) \sum_b \frac{(2S_b+1)\nu_{ab}}{\pi} \int\limits_0^\infty \frac{q^2 f_b^0(q)}{\sqrt{m_b^2+q^2}} \frac{dq}{\Lambda(\bar{s})}, \tag{20}$$

where

$$\bar{s} = \frac{1}{2}pq^4,$$

ν_{ab} is a number of channels of reactions, in which a sort particles can participate a.

Let us calculate values of the integral (20) in extreme cases.

A scattering on nonrelativistic particles. If b sort equilibrium particles are nonrelativistic, i.e., $q \ll m_b$, the integral (20) is reduced to the expression:

$$J_a(p)|_{p\geqslant p_0} =$$

$$= -8\pi^2 \Delta f_a(p) \sum_b \frac{n_b^0(t)}{m_b} \frac{\nu_{ab}}{1+\ln^2 \dfrac{pm_b}{2}}, \quad (m_b > T). \tag{21}$$

A scattering on ultrarelativistic particles. If b sort equilibrium particles are ultrarelativistic, i.e., $m_b \ll T$, moreover their chemical potential is small, - $\mu_b \ll T$, then calculating the integral (20) with respect to the equilibrium distribution (16), we find:

$$J_a(p)|_{p\geqslant p_0} =$$

$$= -\frac{\pi}{3}\frac{\tilde{N}T^2(t)}{1+\ln^2 Tp/2}\Delta f_a(p), \quad (m_b \ll T,\ \mu_b \ll T), \tag{22}$$

where

$$\tilde{\mathcal{N}} = \frac{1}{2}\left[\sum_B (2S+1) + \frac{1}{2}\sum_F (2S+1)\right] = N_B + \frac{1}{2}N_F;$$

N_B is a number of sorts of equilibrium bosons, F - fermions.

To estimate contributions of nonrelativstic and relativistic particles to the integral of collisions in an equilibrium component, we first will calculate their concentrations. A concentration of ultrarelativistic particles in the hot model is resulted from the expression (16) for the distribution function of an equilibrium component by means of the substitution

$$E(p) = p; \quad \mu_a = 0 \tag{23}$$

into the formula for the determination of particles' number density (see [1]):

$$n_a(t) = \frac{2S_a+1}{2\pi^2}\int_0^\infty f_a(t,p)p^2 dp. \tag{24}$$

Thus we find (see [1]):

$$n_a(t) = \frac{(2S_a+1)T^3}{\pi^2} g_n \zeta(3) \tag{25}$$

A concentration of non-relativistic equilibrium particles *at under the assumption of conservation of their number* is violated in proportion to $a^{-3}(t)$. Therefore in

conditions of equilibrium's weak violation the ratio of nonrelativistic particles' density to density of relict photons is approximately constant (since $T \sim a(t)^{-1}$:

$$\frac{n_0(t)}{n_\gamma(t)} \approx \mathrm{Const} = \delta \sim 10^{-10} \div 10^{-9}. \tag{26}$$

Calculating the ratio of contributions to the integral of nonrelativistic and ultrarelativistic particles' collisions, we obtain:

$$J_{non}/J_{ultra} \sim \frac{24\pi n_b^0}{m_b T^2} = \zeta(3)\delta\frac{64T(t)}{\pi m_b} \sim 10^{-9}\frac{T}{m_b}, \tag{27}$$

- ratio of contributions is small at $T \ll 10^9 m_b$ and diminishes with time. Therefore in future we will neglect the contribution of nonrelativistic particles in the integral of collisions.

3 A Relaxation Of A Superthermal Component On Equilibrium Particles

Substituting the received expression (22) for the integral of collisions into the kinetic equations (4), we obtain a kinetic equation, which describes an evolution of an ultrarelativistic superthermal component in the equilibrium cosmological plasma:

$$p\left(\frac{\partial\Delta f_a}{\partial t} - \frac{\dot a}{a}p\frac{\partial\Delta f_a}{\partial p}\right) = -\frac{\pi\tilde N}{3}\frac{T^2(t)}{1+\ln^2 pT/2}\Delta f_a. \tag{28}$$

Taking into account the fact, that the variable:

$$\mathcal{P} = a(t)p, \tag{29}$$

is an integral of motion [4], we proceed to variables $t, \mathcal{P}$ in the equation (28); for any function $\Psi(t,p)$ at that the following relation takes place:

$$\frac{\partial\Psi(t,p)}{\partial t} - \frac{\dot a}{a}p\frac{\partial\Psi(t,p)}{\partial p} = \frac{\partial\Psi(t,\mathcal{P})}{\partial t}. \tag{30}$$

At such a substitution the kinetic equation (28) can be easily integrated in quadratures. For convenience in future we will define more exactly the normalization of the variable $\mathcal{P}$. At that there appears a necessity to compare values, which are used in the nonequlibrium model, with corresponding values of the standard cosmological scenario, since all observed cosmological parameters are interpreted in SCS terms. We also should keep in mind two *synchronous* models of Universe: the real - nonequilibrium model $\mathcal{M}$ with macroscopic parameters $P(t)$ and the ideal - equilibrium model $\mathcal{M}_0$, which in given point of time t possesses certain macroscopic parameters $P_0(t)$.

An Ultrarelativistic Universe

Let us consider the Universe with the ultrarelativistic equation of state[2] (a characteristic of a barotropic line is $\rho = 1/3$):

$$\varepsilon = 3p. \tag{31}$$

Then according to Einstein's equations an energy density of the Universe is changed by law:

$$\varepsilon a^4 = \mathrm{Const}; \qquad \varepsilon = \frac{1}{32\pi t^2}, \tag{32}$$

and a scale factor is changed by law:

$$a(t) \sim t^{1/2}. \tag{33}$$

From the other hand, an energy density of an equilibrium plasma is determined via its temperature by the relation (see [1]):

$$\varepsilon_0 = \mathcal{N}\frac{\pi^2 T^4}{15}. \tag{34}$$

Therefore, if the Universe was filled up *only* by equilibrium plasma, its temperature $T_0(t)$ would change by law (see [1]):

$$\mathcal{N}^{1/4} T_0(t) = \left(\frac{45}{32\pi^3}\right)^{1/4} t^{-1/2} \quad (\sim a^{-1}), \tag{35}$$

- here we take into account a possible weak dependence of an effective number of equilibrium types of particles from time, $\mathcal{N}(t)$. So, let us define more exactly the formula (29) by following way:

$$p = \mathcal{P}\mathcal{N}^{1/4} T_0(t). \tag{36}$$

Such is the meaning of the momentum variable $\mathcal{P}$ according to this formula: *to within the numerical factor of order of one $\mathcal{P}$ is the relation of particles' energy to their average energy in the same point of time in the locally equilibrium ultrarelativistic Universe.*

Thus, solving the kinetic equation (28) with the account of relations (30) and (36), we find its solution:

$$\Delta f_a(t,\mathcal{P}) = \Delta f_a^0(\mathcal{P}) \exp\left[-\frac{\xi(t,\mathcal{P})}{\mathcal{P}} \int\limits_0^t \frac{y^2(t')dt'}{\sqrt{t'}}\right], \tag{37}$$

where:

$$\Delta f_a^0(\mathcal{P}) = \Delta f_a(0,\mathcal{P}),$$

[2] It should be noted, that pressure and momentum have the same denotations.

is an initial deviation from the equilibrium and there is incorporated the dimensionless function:

$$y(t) = \frac{T(t)}{T_0(t)} \tag{38}$$

and the parameter, weakly dependent from variables $t, \mathcal{P}$:

$$\xi(t,\mathcal{P}) = \frac{\pi\tilde{\mathcal{N}}}{3\sqrt{\mathcal{N}}}\left(\frac{45}{32\pi^3}\right)^{1/4}\frac{1}{\Lambda(\mathcal{P}TT_0/2)}; \tag{39}$$

$$\Lambda(x) = 1 + \ln^2 x. \tag{40}$$

Values $\mathcal{P} \gg 1$ correspond to the approximation $p \gg p_T \approx T(t)$.

Since $T(t)$ is a temperature of the equilibrium component of plasma, and $T_0(t)$ is a temperature of the completely equilibrium in a given point of time Universe, the following condition is always fulfilled:

$$y(t) \leqslant 1. \tag{41}$$

To be the correct solution of the kinetic equations, the function $\Delta f_a(t,\mathcal{P})$ has to satisfy in all times to the integral condition (15). Since according to the solution (36) the distribution function's $\Delta f_a(t,\mathcal{P})$ deviation from the equilibrium strictly diminishes with time, for the validity of the solution (36) it is sufficient to the function $\Delta f_a(t,\mathcal{P})$ to satisfy the condition (15) in the initial point of time. This gives:

$$\int\limits_0^\infty \Delta f_a^0(\mathcal{P})\mathcal{P}^2 d\mathcal{P} \gg \frac{2}{\mathcal{N}^{3/4}} y_0^3, \tag{42}$$

where $y_0 = y(0) \leqslant 1$.

As an example we will consider a relaxation of a superthermal component at the initial distribution in form of a staircase function for the density of particles' number:

$$\Delta f^0(\mathcal{P}) = \begin{cases} \dfrac{\pi^2\Delta\tilde{N}}{\mathcal{P}_0\mathcal{P}^2}, & \mathcal{P} \leqslant \mathcal{P}_0; \\ 0, & \mathcal{P}\,\mathcal{P}_0; \end{cases}, \tag{43}$$

so that:

$$\Delta\tilde{N} = \frac{1}{\pi^2}\int\limits_0^\infty \Delta f^0(\mathcal{P})\mathcal{P}^2 d\mathcal{P} \tag{44}$$

is an initial conformal density of the non-equilibrium particles' number. On Fig. 2 an evolution of the superthermal component for such a distribution, at that we laid $y(t) \equiv 1$ is shown .

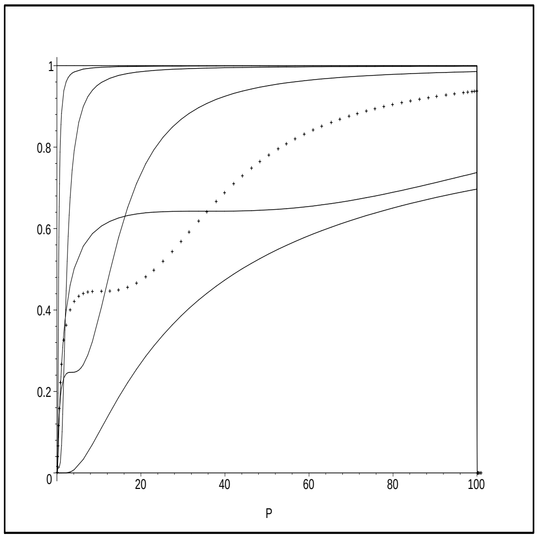

Figure 2: *A relaxation of a superthermal component for the distribution (43) with the assumption $y(t) = 1$ at $\mathcal{P}_0 = 100$, $\tilde{\mathcal{N}}/\sqrt{\mathcal{N}} = 10$. A relative magnitude of the distribution function of particles' number density by conformal energies $\mathcal{P}$ is shown. Top-down (along the figure's left border) there are firm lines: $t = 0$, $t = 0,01$, $t = 0,1$, $t = 1$, $t = 10$ and $t = 100000$; a dotted line is $t = 3$. Time is measured in seconds.*

Let us remind that cosmological time t is measured in Planck units. Therefore a native question, if methods of classical (nonquantized) kinetics can be used in times of order of several Planck times, appears. A condition of applicability of a particles' semi-classical description in the cosmological situation is a relation, resulting from the Heisenberg's uncertainty relation:

$$Et \gg 1. \tag{45}$$

According to (35) and (36):

$$E = p = \mathcal{P}\left(\frac{45}{32\pi^3}\right)^{1/4} t^{-1/2}. \tag{46}$$

Therefore a condition of applicability of a particles' semi-classical description (45) takes form:

$$t\mathcal{P}^2 \gg \sqrt{\frac{32\pi^2}{45}} \approx 2,65. \tag{47}$$

Such consideration at $t \sim 1$ is justified for sufficiently great values of the conformal momentum $\mathcal{P} \gg 1$, which exactly correspond superthermal particles. Thus, *a semi-classical description of particles is applicable in Planck times of an evolution of the Universe with the more validity the lesser is a ratio of a*

thermal energy to a particle's energy. Thus, a description of a superthermal (non-equilibrium) component's evolution does not require a quantum consideration.

Non-equilibrium Universe

Let us consider now the Universe on non-relativistic stage of expansion. In this case an equation of state is

$$p = 0 \tag{48}$$

and a scale factor is changed by law:

$$a(t) \sim t^{2/3}. \tag{49}$$

We will consider at that a scattering of superthermal particles on equilibrium massless relic particles, a number of which in SCS is greater than a number of non-relic particles approximately in 10^9. Let t_0 is a moment of an ultrarelativistic equation of state's replacement by a non-relativistic one in the non-equilibrium Universe and $T^0_\gamma(t)$ is a temperature of relic photons in the equilibrium Universe. Then:

$$T^0_\gamma(t) = \left(\frac{45}{32\pi^3\mathcal{N}}\right)^{1/4} \frac{t_0^{1/6}}{t^{2/3}}. \tag{50}$$

Integrating the kinetic equation (28), we find in this case:

$$\Delta f_a(t,\mathcal{P}) = \Delta f_a(t_0,\mathcal{P}) \exp\left[-\frac{\chi(t,\mathcal{P})}{\mathcal{P}} \int\limits_{t_0}^{t} \frac{y^2(t')dt'}{t'^{2/3}}\right], \tag{51}$$

where $\Delta f_a(t_0,\mathcal{P})$ is determined by the solution (37), and

$$\chi(t,\mathcal{P}) = \frac{\pi\tilde{\mathcal{N}} T^0_\gamma(t_0) t_0^{2/3}}{\Lambda(\frac{1}{2}\mathcal{P} T^0_\gamma(t))} \tag{52}$$

is a slowly changing parameter, $\tilde{\mathcal{N}}$ is determined for massless relic particles.

4 A Heating Of An Equilibrium Component By Superthermal Particles

4.1 An Energy-balance Equation

In spite of a small in comparison with an equilibrium number of non-equilibrium particles, energy, included in the non-equilibrium tail, can turn to be sufficiently greater than an energy of an equilibrium component, if sufficient distortions of the distribution lie in a superthermal range, to which great values of the momentum variable $\mathcal{P} \gg 1$ correspond. Superthermal particles, colliding with

equilibrium ones, transfer theirs energy to equilibrium particles and thereby heat up an equilibrium component of plasma. For the finding of the real temperature, $T(t)$, of an ultrarelativistic plasma with the account of its heating by superthermal particles we will use the equation (32), which presents itself a consequence of the energy's conservation law and determines the dependence of the ultrarelativistic Universe's energy density, ε, from the cosmological time. This energy density is composed from the energy density of an equilibrium plasma, ε_0 (33), and the energy density of a superthermal component, ε_1:

$$\varepsilon = \varepsilon_0 + \varepsilon_1, \tag{53}$$

where

$$\varepsilon_1 = \frac{\mathcal{N}T_0^4}{\pi^2}\sum_a (2S_a+1)\int\limits_0^\infty \Delta f_a(t,\mathcal{P})\mathcal{P}^3 d\mathcal{P}. \tag{54}$$

For the further it is convenient to incorporate a dimensionless variable $\sigma(t)$:

$$\sigma(t) = \frac{\varepsilon_0}{\varepsilon} = \frac{\varepsilon_0}{\varepsilon_0+\varepsilon_1} \leqslant 1. \tag{55}$$

Since the total energy density $\varepsilon(t)$ is determined from the other hand via fourth degree of temperature, $T_0(t)$, of the completely equilibrium Universe, and an energy density of an equilibrium component $\varepsilon_0(t)$ is determined via fourth degree of temperature $T(t)$, an incorporated dimensionless variable is concerned by a simple relation with a dimensionless variable $y(t) = T(t)/T_0(t)$, incorporated earlier (38):

$$\sigma(t) = y^4(t), \tag{56}$$

from which right away the inequality (41) follows.

Thus from (53) subject to the solution (37) for the non-equilibrium distribution function we obtain an integral equation relative to function $y(t)$:

$$y^4 + \frac{15}{\pi^4}\sum_a (2S_a+1)\int\limits_0^\infty \mathcal{P}^3 \Delta f_a^0(\mathcal{P})\times$$

$$\times \exp\left[-\frac{\xi(t,\mathcal{P})}{\mathcal{P}}\int\limits_0^t \frac{y^2(t')dt'}{\sqrt{t'}}\right] d\mathcal{P} = 1. \tag{57}$$

From this equation in zero point of time we obtain the relation:

$$\frac{15}{\pi^4}\sum_a (2S_a+1)\int\limits_0^\infty \mathcal{P}^3\Delta f_a^0(\mathcal{P})d\mathcal{P} = 1-\sigma_0. \tag{58}$$

4.2 A Solution Of An Energy-balance Equation

At specified functions $\Delta f_a^0(\mathcal{P})$ an equation (57) is always integrated with a logarithmic accuracy in quadratures. Actually, instead of variable t and function $y(t)$ we incorporate new dimensionless variable τ:

$$\tau = \frac{\overline{\xi}}{\overline{\mathcal{P}}_0}\sqrt{t} \tag{59}$$

and function $Z(\tau)$:

$$Z(\tau) = 2\int\limits_0^\tau y^2(\tau')d\tau', \tag{60}$$

where $\overline{\xi} = \xi(\overline{\mathcal{P}}_0)$,

$$\overline{\mathcal{P}}_0 = \frac{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\mathcal{P}\mathcal{P}^3\Delta f_a^0(\mathcal{P})}{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\mathcal{P}\mathcal{P}^2\Delta f_a^0(\mathcal{P})} \tag{61}$$

is an average value of the momentum variable $\mathcal{P}$ in the point of time $t = 0$. At that we obtain:

$$y = \sqrt{\frac{1}{2}Z'_\tau}; \quad Z(0) = 0; \quad Z'_\tau(0) = 2\sqrt{\sigma_0}, \tag{62}$$

where $\sigma_0 = \sigma(0) = y^2(0)$. Then after an integral's calculation with a logarithmic accuracy the equation (57) subject to the relation (58) is reduced to the form [3]:

$$Z'_\tau = 2\sqrt{1-(1-\sigma_0)\Phi(Z)}, \tag{63}$$

where function $\Phi(Z)$ is incorporated:

$$\Phi(Z) = \frac{\overline{\mathcal{P}}(t)}{\overline{\mathcal{P}}(0)} =$$

$$= \frac{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\mathcal{P}\mathcal{P}^3\Delta f_a^0(\mathcal{P})e^{-Z\overline{\mathcal{P}}_0/\mathcal{P}}}{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\mathcal{P}\mathcal{P}^3\Delta f_a^0(\mathcal{P})}. \tag{64}$$

Henceforth it will be convenient to proceed to a new dimensionless momentum variable:

$$\rho = \frac{\mathcal{P}}{\overline{\mathcal{P}}_0}, \tag{65}$$

such that:

$$1 = \frac{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\rho\rho^3\Delta f_a^0(\rho)}{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\rho\rho^2\Delta f_a^0(\rho)} \Rightarrow \overline{\rho}_0 \equiv 1. \tag{66}$$

Then:

$$\Phi(Z) = \frac{\overline{\rho}(\tau)}{\overline{\rho}(0)} =$$

$$= \frac{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\rho\rho^3\Delta f_a^0(\rho)e^{-Z/\rho}}{\sum\limits_a (2S_a+1)\int\limits_0^\infty d\rho\rho^3\Delta f_a^0(\rho)}. \tag{67}$$

It is obvious, that $\Phi(Z)$ is a monotone decreasing function of Z, since it is always fair

$$\Phi'_Z < 0, \qquad Z \in (0,\infty), \tag{68}$$

at that $\Phi(0) = 1$ and $\Phi(\infty) = 0$. From this it follows:

$$0 \leqslant \Phi(Z) \leqslant 1. \tag{69}$$

From the definition (67) it also follows, that the following relation is always fair

$$\Phi''_{ZZ} > 0, \qquad (Z \in (0,\infty)), \tag{70}$$

therefore graph of function $\Phi(Z)$ is concave. In consequence of a strict monotonicity and continuous differentiability of function $\Phi(Z)$ according to the equation (66) function $Z(\tau)$ with its first derivative are monotone increasing functions of the variable x. Integrating the equation (66), we obtain:

$$\frac{1}{2}\int\limits_0^Z \frac{dU}{\sqrt{1-(1-\sigma_0)\Phi(U)}} = \tau. \tag{71}$$

Stated monotonicity properties of functions $\Phi(Z)$ and $Z(\tau)$ guarantee us the equation (71) at each specified value τ has the unique solution $Z(\tau)$. At a computed value of function $Z(\tau)$ a value of temperature $T(t)$ (or $y(t)$) of an equilibrium component can be obtained according to (65) and (66) by means of the formula:

$$y = [1-(1-\sigma_0)\Phi(Z)]^{1/4}. \tag{72}$$

Thus, the problem of heating of an equilibrium component of plasma is formally solved.

It should be noted, that a condition of approximation's applicability of the LTE's weak violation (42) in terms of incorporated here values $\overline{\mathcal{P}}_0$ and σ_0 takes the form:

$$\mathcal{N}^{1/4}\sigma_0^{3/4}\overline{\mathcal{P}}_0 \gg 1-\sigma_0. \tag{73}$$

4.3 An analysis of a thermal heating's process of establishment

Let us proceed now to the analysis of the obtained solution.

An asymptotic behavior at small times

First we will consider an asymptotic behavior of solutions at small cosmological times:

$$t \to 0 \Rightarrow \tau \ll 1. \tag{74}$$

Expanding a subintegral expression in the right side of (67) in Taylor's series by the degrees of smallness of Z, we obtain subject to the definition (64) an asymptotic expansion of function $\Phi(Z)$:

$$\Phi(Z) = 1 - Z + O^2(Z). \tag{75}$$

Substituting (75) into the equation (63) and integrating the obtained equation subject to initial conditions (62), we find an asymptotic, at small times, solution:

$$Z = \tau\sqrt{\sigma_0} + \tau^2(1-\sigma_0), \tag{76}$$

from which subject to (62) we obtain:

$$y(t) = \sqrt{\sqrt{\sigma_0} + \tau(1-\sigma_0)}. \tag{77}$$

The case $\sigma_0 \ll 1$ corresponds to a small energy density of an equilibrium component in comparison with an energy density of a non-equilibrium component of plasma in the point of time $t = 0$. According to (73) a consideration of this case is justified for sufficiently great values of $\overline{\mathcal{P}}_0$:

$$\overline{\mathcal{P}}_0 \gg \sigma_0^{-3/4}. \tag{78}$$

In this case we obtain from (77) an asymptotic law of temperature's variation in the early Universe:

$$T(t) = T_\gamma^0(t)\left(\frac{\overline{\xi}}{\overline{\overline{\mathcal{P}}}_0}\right)^{1/2} t^{1/4} \sim t^{-1/4} \tag{79}$$

a temperature of plasma falls more slowly than in SCS, but in each point of time the real temperature is lower than the corresponding temperature in SCS:

$$y(t) \leqslant 1 \Rightarrow T(t) \leqslant T_0(t), \tag{80}$$

and is compared with the last in a certain point of time $\overline{t}$. Let us find this time. Substituting (79) into the solution (37), we find a law of evolution of the superthermal particles' distribution in the Universe's early stage:

$$\Delta f_a(t,\mathcal{P}) = \Delta f_a^0(\mathcal{P}) \exp\left(-\frac{\overline{\xi}^{\,2} t}{\overline{\overline{\mathcal{P}}}_0 \mathcal{P}}\right). \tag{81}$$

According to (79) and (81) LTE as a whole is recovered in times

$$t > \overline{t} = \left(\frac{\overline{\mathcal{P}}_0}{\overline{\xi}}\right)^2, \tag{82}$$

is exactly a time of an equilibrium's establishment. A plasma's temperature at that reaches its equilibrium value $T_0(t)$ (constantly decreasing at that with time). However, for particles with energies higher than the average one $\overline{\mathcal{P}}_0$, the equilibrium is not reached yet in this time.

An asymptotic behavior at great times in the ultrarelativistic Universe

Supposing $t > \overline{t}$ and, consequently, $y(t) \approx 1$, we will obtain an asymptotic form (37) on great times:

$$\Delta f_a(t, \mathcal{P}) = \Delta f_a^0(\mathcal{P}) \exp\left(-2\frac{\xi(\mathcal{P})\sqrt{t}}{\mathcal{P}}\right); \tag{83}$$

thus, thermal heating's establishment time for particles with the momentum $\mathcal{P}$ is:

$$t_{\mathcal{P}} = \left[\frac{\mathcal{P}}{\xi(t, \mathcal{P})}\right]^2 \approx \left(\frac{\mathcal{P}}{\overline{\mathcal{P}}_0}\right)^2 t. \tag{84}$$

Let us suppose, that in range of great values of $\mathcal{P}$ the initial distribution of superthermal particles is extrapolated via power law:

$$\Delta f_a^0(\mathcal{P}) \sim \mathcal{P}^{-\lambda}; \quad (\mathcal{P} \gg 1;\ \lambda > 4). \tag{85}$$

Then an average energy of superthermal particles $\overline{E}_1(t)$, for which a thermal equilibrium to moment of time t is not reached yet, is equal:

$$\overline{E}_1(t) = \xi(t, \mathcal{P}) T_0(t) \sqrt{t} \approx \frac{1}{\Lambda(T_0^2(t)\overline{\mathcal{P}}_0)} \sim \mathrm{Const} \tag{86}$$

practically does not depend on time. In the previous paper [1] we noted that in the modern stage $1/\Lambda \sim \alpha^2$, where α is a fine structure constant. Thus, we obtain from (86) the estimation:

$$\overline{E}_1(t) \sim 10^{-4} \tag{87}$$

is in ordinary units $\overline{E}_1(t) \sim 10^{15}$ Gev - i.e, particles with an energy of order of a grand unification character energy or higher stay non-equilibrium. This conclusion is fundamentally important for cosmological scenarios.

An asymptotic behavior at great times in the non-relativistic Universe

Let us consider now an evolution of superthermal ultrarelativistic particles in the non-relativistic Universe, the thermal equilibrium in which on average is already recovered. Supposing in this case $y = 1$ in the expression (51) for the distribution function of superthermal particles, we obtain:

$$\Delta f_a(t,\mathcal{P}) = \Delta f_a(t_0,\mathcal{P})\times$$

$$\times \exp\left\{-\frac{3(\pi\mathcal{N})^{1/4}\chi(\mathcal{P})}{\mathcal{P}}\sqrt{t_0}\left[\left(\frac{t}{t_0}\right)^{1/3}-1\right]\right\}, \tag{88}$$

the distribution of superthermal particles evolves more slowly, than (83). A time of the establishment of the thermal equilibrium for particles with momentum $\mathcal{P}$ in this case is equal:

$$t_{\mathcal{P}} \simeq \frac{1}{\sqrt{t_0}}\left[\frac{\mathcal{P}}{\xi(t,\mathcal{P})}\right]^3, \tag{89}$$

and an average energy os superthermal particles falls slowly with time:

$$\overline{E}_1(t) \simeq \frac{\overline{\mathcal{N}}}{\sqrt{\mathcal{N}}\Lambda(T_0^2(t)\mathcal{P}/2)}\left(\frac{t_0}{t}\right)^{1/3} \sim \tag{90}$$

$$\sim 10^{15}\left(\frac{t_0}{t}\right)^{1/3}\text{Gev.}$$

In the modern age this value is of order $10^{12} \div 10^{13}$Gev at variation of t_0 in limits $10^9 \div 10^{12}$sec.

Numerical model

Let us proceed now from estimations to numerical calculations. Let us present an initial deviation of the distribution function from the equilibrium in the form:

$$\Delta f^0(x) = \frac{A}{\mathcal{P}_0^3(k^2+x^2)^{3/2}}\chi(1-x), \quad k \to 0, \tag{91}$$

where $\chi(z)$ is a Heaviside's function (a staircase function), $x = \mathcal{P}/\mathcal{P}_0$ is a dimensionless momentum variable, A, $\mathcal{P}_0$ and k are certain parameters; a parameter k is introduced for the security of convergence of all distribution function's moments in the range of momentum's small values. The distribution function of superthermal particles' energy density at that has a form, similar to the spectrum of so-called *flat noise*, when all values of energy are equiprobable (see Fig. 3 and [10]).

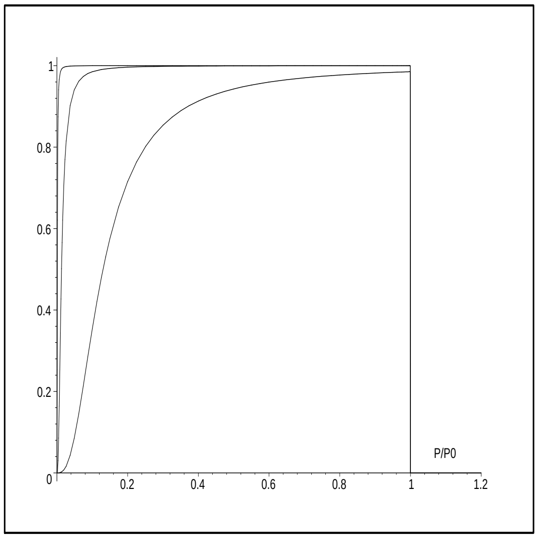

Figure 3: *An initial deviation of the distribution function of the superthermal particles' energy $\Delta f^0(x)x^3$ from the equilibrium by the formula (91), bottom-up is: $k = 0,1$, $k = 0,01$, $k = 0,001$.*

Calculating a number density of particles relatively to the distribution (91), we find according to (24):

$$n_1(t) =$$
$$= \frac{A(2S_a+1)\mathcal{N}^{3/4}}{2\pi^2} T_0^3(t) \left(\ln \frac{k}{\sqrt{1+k^2}-1} - \frac{1}{\sqrt{1+k^2}} \right).$$

Thus, we obtain in the extreme case:

$$n_1(t) \simeq A \frac{(2S_a+1)\mathcal{N}^{3/4}}{2\pi^2} T_0^3(t) \ln \frac{2}{k} \qquad (k \to 0).$$

from which follows, that for a fulfilment of the condition (15) in all times it is necessary and sufficiently the following relation to be fulfilled:

$$A\mathcal{N}^{3/4} \ln \frac{2}{k} \ll 2\sigma_0^{3/2} \zeta(3).$$

Calculating now an energy density relatively to the distribution (91), we obtain:

$$\varepsilon_1(t) = A \frac{\mathcal{N} T_0^4(t)(2S+1)}{\pi^2} \mathcal{P}_0 (1-k)^2.$$

Thus, subject to (36) we obtain a dependence of a momentum variable's average value through parameters $\mathcal{P}_0$ and k (Fig. 4):

$$\overline{\mathcal{P}}_0 = \mathcal{P}_0 \frac{(1-k)^2}{\ln \dfrac{k}{\sqrt{1+k^2}-1} - \dfrac{1}{\sqrt{1+k^2}}}$$

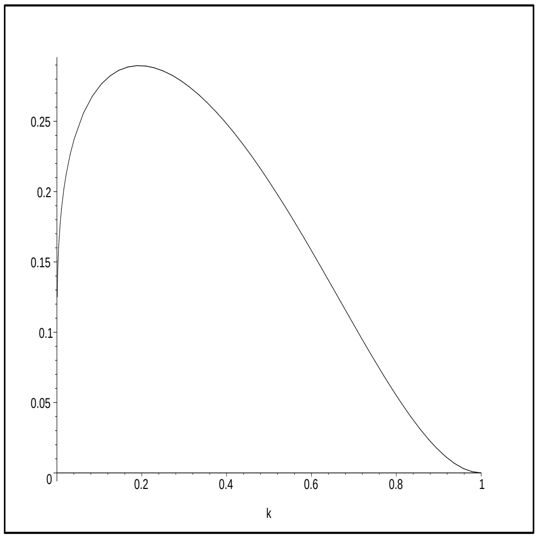

Figure 4: *A dependence of variable's relative average $\overline{\mathcal{P}}_0/\mathcal{P}_0$ from the parameter k of the distribution (91).*

Then an energy-balance equation (53) or (57) at $t \to 0$ leads to the relation between the distribution parameters (91):

$$\frac{15}{\pi^4}A(2S+1)\mathcal{P}_0(1-k)^2 = 1-\sigma_0. \tag{92}$$

Thus, the following condition, imposing a restriction on model's parameters, has to be fulfilled:

$$A\mathcal{P}_0 < \frac{\pi^4}{30}, \tag{93}$$

Calculating the function $\Phi(Z)$ relatively to the distribution (91) according to the formula (67), we find, proceeding to the limit $k \to 0$:

$$\Phi(Z) = e^{-Z} + \mathrm{Ei}(-Z), \tag{94}$$

where $\mathrm{Ei}(z)$ is an integral exponential function [11]:

$$\mathrm{Ei}(z) = \int\limits_{-\infty}^{z} \frac{e^t}{t}dt, \quad |\arg(-z)| < \pi. \tag{95}$$

The equation (71) accounting the relation (95) takes the form:

$$\tau = \frac{1}{2}\int\limits_0^\theta \frac{du}{\sqrt{1-(1-\sigma_0)(\exp(-u)+u\mathrm{Ei}(-u))}}, \tag{96}$$

where $\tau = x/\mathcal{P}_0$. If we could integrate (96) and found the dependence $\theta(\tau) \to Z(x)$, we could receive the required dependence $y(x)$ from the relations (62)-(63) in the form:

$$y = \Big[1 - (1-\sigma_0)(e^{-\theta(\tau)} + \theta(\tau)\mathrm{Ei}(-\theta(\tau))\Big]^{1/4}. \tag{97}$$

However, the dependence $\theta(\tau)$ of course, can not be found. Therefore we act in the following way: we find the dependence $\tau(\theta)$ from the equation (96) by means of the direct numerical integration. Then the equation (97) can be considered as an equation of type $y = y(\theta)$, and the set of two equations (96) and (97) - as parametric equations of the graph $y = y(\tau)$:

$$\begin{aligned} \tau(\theta) &= \frac{1}{2}\int\limits_0^\theta \frac{du}{\sqrt{1-(1-\sigma_0)(\exp(-u) + u\mathrm{Ei}(-u))}}, \\ y(\theta) &= \Big[1 - (1-\sigma_0)(e^{-\theta(\tau)} + \theta(\tau)\mathrm{Ei}(-\theta(\tau))\Big]^{1/4} \end{aligned} \tag{98}$$

On Fig. 6 the results of a numerical modelling of the equilibrium component's heating process by superthermal particles for the initial distribution (91) on the basis of equations (98) are shown.

From this figure it is obvious that at $\tau \sim 1$ temperature of plasma's equilibrium component in fact becomes saturated up to the value $T_0(t)$, i.e., to the point of time $t \sim \mathcal{P}_0^2/\overline{\xi}^2$, that coincides with the asymptotic estimation (82).

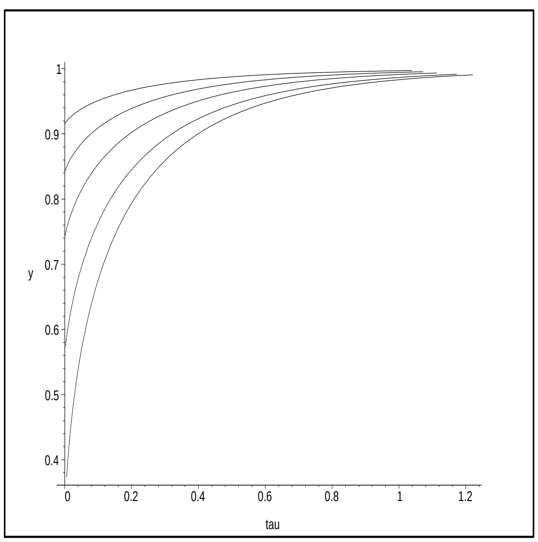

Figure 5: *A relaxation of plasma's temperature to the equilibrium: $y = T(t)/T_0(t)$ subject to the parameter σ_0: bottom-up*

$\sigma_0 = 0,01;\ 0,1;\ 0,3;\ 0,5;\ 0,7$. Values of the dimensionless time variable τ are positioned on the abscissa axis.

5 Basic regularities of a cosmological model with an initially weakly violated thermal equilibrium

5.1 Concentrations of ultrarelativistic relic particles

Equilibrium concentrations of ultrarelativistic particles in the Universe with a weakly violated thermodynamical equilibrium are determined by the relation (25), which subject to the definition (38) can be rewritten in terms of the incorporated function $y(t)$:

$$n_a(t) = \frac{\rho T^3(t)}{\pi^2} g_n \zeta(3) = n_a^0(t) y^3(t) \leqslant n_a^0(t), \tag{99}$$

where $n_a^0(t)$ are the equilibrium concentrations of the same particles in SCS:

$$n_a^0(t) = \frac{\rho T_0^3(t)}{\pi^2} g_n \zeta(3) =$$

$$= \frac{\rho}{\pi^2} g_n \zeta(3) \left(\frac{45}{3\pi^3 \mathcal{N}}\right)^{3/4} t^{-3/2}. \tag{100}$$

Thus, the number of ultrarelativistic particles lying in the equilibrium in each given point of time t is lesser than in SCS. If the interaction between particles is described by the scaling cross-section of type (1) or (2), than a number of non-equilibrium particles is described by the distribution function of type (37). Let us consider particles' reactions in the range of low energies, in which scaling can be violated. Let $\tau^a_{eff}(t)$ is an effective time of interactions of sort "a" particles with the other particles and let in the investigated range of energies the inequality:

$$\tau^a_{eff}(t) > t \tag{101}$$

has as its solution:

$$t > t_a^*. \tag{102}$$

Then in the point of time $t = t_a^*$ sort "a" particles cease to interact with the other, i.e., become *relic* — the number of such stable particles in times, later than t_a^*, is conserved, but their density evolves henceforth by law:

$$n_a(t) = n_a^0(t_a^*) y^3(t_a^*) \left[\frac{a(t_a^*)}{a(t)}\right]^3. \tag{103}$$

Therefore, if in the point of time $t = t_a^*$ an equilibrium is not recovered as a whole, the number of relic "a" sort particles is in $y^3(t_a^*)$ smaller, than it

is received in the standard scenario[3]. Thus, varying a parameter of the non-equilibrium Universe's model $\overline{\mathcal{P}}_0$ at $\sigma_0 \ll 0$ we can regulate the number of relic particles and make this number an arbitrarily small [8] - at a increase of $\overline{\mathcal{P}}_0$ the number of relic particles decreases. Thus, at $\overline{\mathcal{P}}_0 > 10^2$ and $\sigma_0 \ll 1$ relic extra-massive bosons disappear, at $\overline{\mathcal{P}}_0 > 3 \cdot 10^{17}$ and $\sigma_0 \ll 1$ relic neutrinos disappear. This approximate estimator, made in papers, [3], [8], will be specified below.

5.2 Relic Neutrinos

So, let us investigate the problem of relic neutrinos' outcome during the hardening process. A thermal equilibrium of electron and muon neutrino is established, generally, by reactions:

$$\left.\begin{array}{lcl} e^- + e^+ & \rightleftarrows & \nu_e + \overline{\nu}_e, \\ \mu^+ + \mu^- & \rightleftarrows & \nu_\mu + \overline{\nu}, \\ \mu^+ & \rightarrow & e^+ + \overline{\nu}_\mu + \nu_e, \\ \mu^- & \rightarrow & e^- + \nu_\mu + \nu_e. \end{array}\right\} \tag{104}$$

The cross-section of an electro-weak interaction, corresponding to reactions of the neutrino annihilation (104) in the interesting for us range of sufficiently low energies E, is described by the expression:

$$\sigma_n u \approx \frac{G_\mu^2 E^2}{\hbar^4 c^4}, \tag{105}$$

where $G_\mu \approx G_n u := 1,4358 \cdot 10^{-49}\mathrm{erg/cm}^3$ is a constant of an electro-weak interaction. Calculating the hardening time of electron neutrinos, t_ν by means of the formula:

$$\tau_{eff} = \frac{1}{n_e(t_\nu)\sigma_\nu(t_\nu)c} = t_\nu, \tag{106}$$

where it is necessary to substitute $E = T(t)$, an expression for an equilibrium density of ultrarelativistic electrons (25) and a temperature of an equilibrium component $T(t)$, calculated above. Then the ratio of the electron neutrinos' number after the hardening in a non-equilibrium model to the same number in an equilibrium one N_ν, is determined by means of the expression:

$$N_\nu = \frac{T(t_\nu)^3}{T_0(t_\nu)^3} = y^{3/4}(t_\nu). \tag{107}$$

On Fig. 6 the results of numerical calculations of electron neutrinos' outcome in a weakly-equilibrium model of the Universeare are shown. These results, in general, confirm referred above qualitative assessments of previous papers.

[3]The time t_a^* is often called the hardening time (see for example [12]).

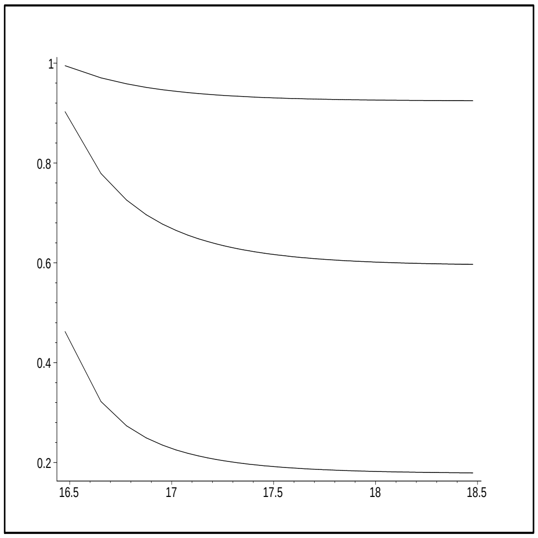

Figure 6: *An outcome of electron relic neutrinos after the hardening relative to a standard model, n_ν/n_ν^0, (an ordinate axis) subject to a parameter of a non-equilibrium model, $lg\mathcal{P}_0$, (an abscissa axis). Top-down:* $\sigma_0 = 0,9$, $\sigma_0 = 0,5$, $\sigma_0 = 0,1$.

5.3 A hardening of neutrinos and cosmological helium's generation

A thermal equilibrium of non-relativistic neutrinos is established in reactions of type:

$$\left.\begin{array}{lcl} e^+ + n & \rightleftarrows & p + \overline{\nu}, \\ \nu + n & \rightleftarrows & p + e^-. \end{array}\right\} \tag{108}$$

A concentration ratio of relic neutrinos, N_n, is determined via the kinetic equation[4]:

$$\frac{dN_n}{dt} + (a+b)N_n = b, \tag{109}$$

where:

$$N_n(t) + N_p(t) = 1, \tag{110}$$

(N_p is a concentration ratio of protons), coefficients $a(t)$ and $b(t)$ (velocities of reactions (108)) are related via the identity:

$$b(t) = a(t)e^{-\Delta mc^2/T(t)}, \tag{111}$$

$\Delta m = m_n - m_p \approx 1,3$Mev is a mass defect of neutron.

[4]Details see, for example, in [12].

Let us investigate the equation (109). At $t \to 0$ according to (111) $a(t) \to b(t)$, therefore at $t \to 0$ from (105) we obtain the relation:

$$\left.\frac{dN_n}{dt}\right|_{t\to 0} = a(t)[1 - 2N_n(0)]. \qquad (112)$$

From (112) right away follows, that at $N_n(0) > 1/2$ the concentration of neutrons from the very beginning decreases with time, and at $N_n(0) < 1/2$ - increases with time. In the second case the function $N_n(t)$ always has a maximum in the point of time t_*, determinable by the relation:

$$N_n(t_*) = \frac{1}{1 + e^{\Delta mc^2/T(t_*)}} < \frac{1}{2}. \qquad (113)$$

At $N_n(0) = 1/2$ developing into the Taylor's series $N_n(t)$ by the small t and using at that the equation (109) and limitary relations at $t \to 0$, we obtain:

$$N_n(t) \approx \frac{1}{2} + \frac{1}{4}\, a(t)\frac{\Delta mc^2}{T^2}\left.\frac{dT}{dt}\right|_{t=0} t^2 + \ldots$$

Since the second member in the right side of the development is the negative one, then at $N_n(0) = 1/2$ the concentration of neutrons decreases.

Let us proceed now to the numerical modelling. The coefficient $a(t)$ in units $\sec^{-1}$ at high temperatures is described by an approximate expression (see for example, [12]):

$$a(t) \approx 1,61 \cdot \tilde{T}^5(t), \qquad (114)$$

where $\tilde{T}(t)$ is a temperature in Mev. At low temperatures $a(t) = W \approx 10^{-3} sec^{-1}$, where W is a probability of free neutron's decay. Supposing in (35) subsequent to [12] $\mathcal{N} = 4,5$, we obtain for $\tilde{T}_0(t)$ the following relation:

$$\tilde{T}_0 = 0,89 t^{-1/2}, \qquad (115)$$

where time is measured in seconds. How the numerical calculations show, this point of time is reached sufficiently fast[5], - the further history of the nucleosynthesis does not depend in fact from the initial concentration ratio of neutrons.

Let us consider reactions (108) on conditions, when the equilibrium in plasma is not reached as a whole, i.e., $y(t) < 1$. Supposing at that function $y(t)$ obey the asymptotic form (77), and at interesting for us time scales $\Lambda \simeq \alpha^{-2} \approx (137)^2$, we obtain:

$$\tilde{T} \approx 0,89\frac{1}{\sqrt{t}}\sqrt{\sqrt{\sigma_0} + 0,523(1 - \sigma_0)\frac{\sqrt{t}}{P_0}}, \qquad (116)$$

where:

$$P_0 = \frac{\mathcal{P}_0}{10^{17}}.$$

On Fig. 7-10 the results of the numerical integration of the equation (116) are shown.

[5]The value of N_n at that in maximum is amount to approximately 0,45.

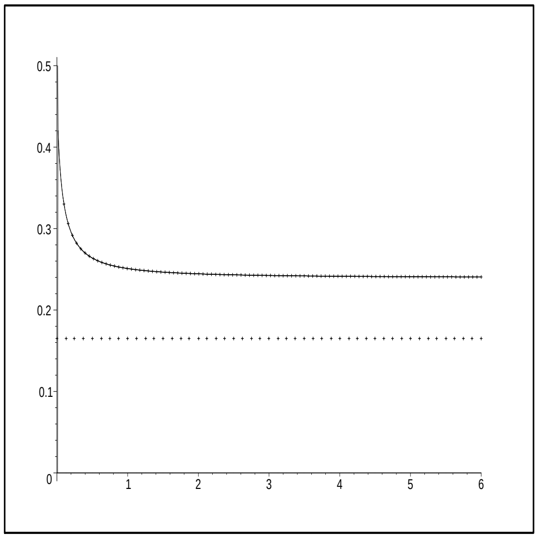

Figure 7: *An influence of the neutrons' initial concentration on nucleosynthesis in the none-equilibrium Universe at parameters' value $\mathcal{P}_0 = 10^{18}$ and $\sigma_0 = 0, 1$: along the abscissa axis time in seconds is placed, along the ordinate axis - the concentration of neutrons, $N_n(t)$ is placed. The thin line is $N_n(0) = 0$, heavy line is $N_n(0) = 0, 5$, dotted line is $N_n(0) = 1$; the value $N_n = 0, 165$ is noted via the dotted straight line. How it is seen from the figure, all three curves practically coincide, differing only for small values of time, $t < 0, 2sec$.*

The results of the numerical integration, shown on Fig. 7, confirm the fact, that the final concentration of neutrons practically does not depend from their initial concentrations. The number of neutrons after the hardening at $\mathcal{P}_0 \lesssim 10^{17}$ practically coincide with the similar number in the hot model (SCS). The final outcome of neutrons after the hardening at $\mathcal{P}_0 > 10^17$ increases with the growth of $\mathcal{P}_0$, however at the constant value of $\mathcal{P}_0$ slowly decreases with the growth of the equilibrium parameter σ_0.

Adduced results of numerical calculations of the neutrons' hardening in the non-equilibrium Universe, confirm, in general, estimations, obtained earlier in papers [3] and [8], merely specifying their in details. Let us note, that a maximal weight concentration of the cosmological He_4, N_{He_4}, is equal to (see [12]):

$$\max(N_{He_4}) = 2N_n. \tag{117}$$

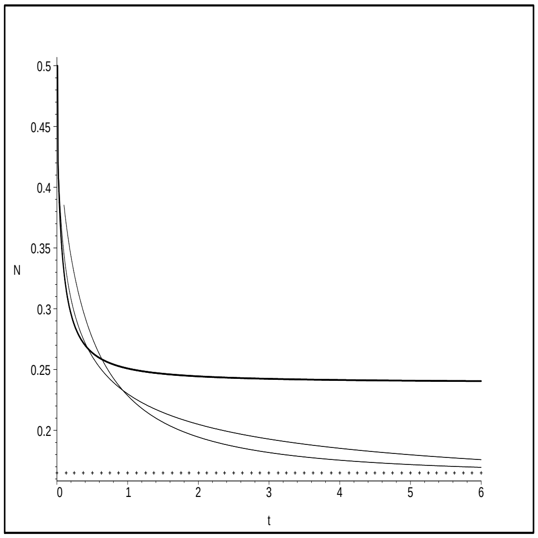

Figure 8: *An influence of the parameter $\mathcal{P}_0$ on the nucleosynthesis in the non-equilibrium Universe subject to the nucleosynthesis in the non-equilibrium Universe at $N_n(0) = 0,5$ and $\sigma_0 = 0,1$: on the abscissa axis time in seconds is placed, on the ordinate axis the concentration of neutrons, $N_n(t)$ is placed. The lower line is a classic result (see [12]), the middle line is $\mathcal{P}_0 = 10^{17}$, the heavy line is $\mathcal{P}_0 = 10^{18}$; by means of the dotted line the value $N_n = 0,165$ is noted.*

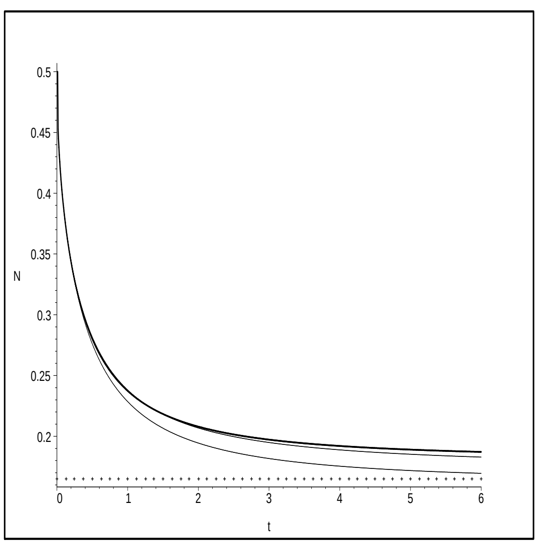

Figure 9: *An influence of the parameter $\mathcal{P}_0$ on the nucleosynthesis in the non-equilibrium Universe at $N_n(0) = 0,5$ and $\sigma_0 = 0,9$: on the abscissa axis*

time in seconds is placed, on the ordinate axis the concentration of neutrons, $N_n(t)$ is placed. The lower line is a classic result (see [12]), the middle line is $\mathcal{P}_0 = 10^{17}$, the heavy line is $\mathcal{P}_0 = 10^{18}$; by means of the dotted line the value $N_n = 0,165$ is noted.

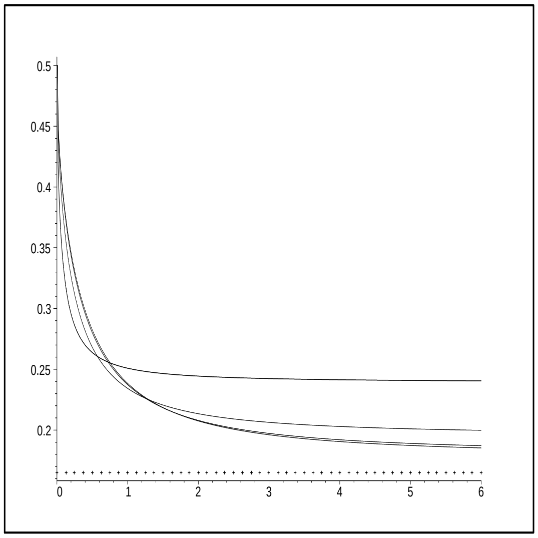

Figure 10: *An influence of the parameter σ_0 on the nucleosynthesis in the non-equilibrium Universe at $N_n(0) = 0,5$ and $\mathcal{P}_0 = 10^{18}$: on the abscissa axis time in seconds is placed, on the ordinate axis the concentration of neutrons, $N_n(t)$ is placed. Top-down: $\sigma_0 = 0,1$, $\sigma_0 = 0,5$, $\sigma_0 = 0,9$, $\sigma_0 = 0,99$; by means of the dotted line the value $N_n = 0,165$ is noted. Last two lines are practically indistinguishable.*

5.4 An observation restrictions on parameters of the non-equilibrium distribution

Owing to the concentration growth of the cosmological He_4 an observation data about the content of He_4 in the Universe on the level not higher than 35% at increase of the parameter $\mathcal{P}_0$ imposes restrictions on upper values of the non-equilibrium model's parameter $\mathcal{P}_0$ at a specified value of its another parameter σ_0. This fact was noted earlier in papers [3], [8], where on the basis of quantitative assessments the range of allowed values of non-equilibrium model's parameters was specified. However, these values, correct at not large values of the parameter σ_0, to be improved in the range $\sigma_0 \sim 1$. Such improved picture is represented on Fig. 11.

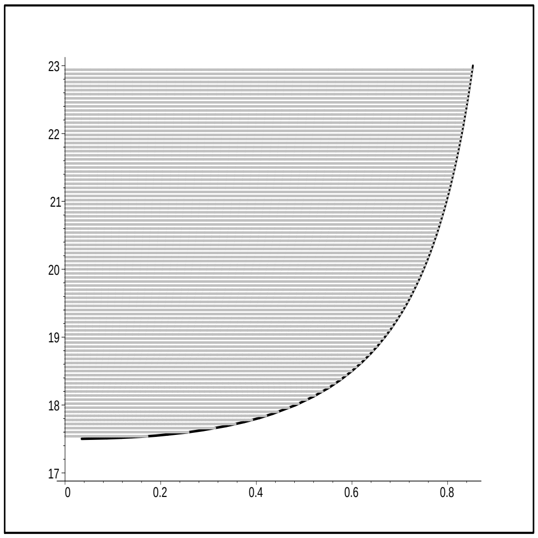

Figure 11: *An allowed by the concentration of He_4 range of weakly-equilibrium model's parameters* $\lg \mathcal{P}_0$ *(an ordinate axis)* σ_0 *(an abscissa axis) - an excluded range of parameters' values is hatched.*

Conclusion

Summing up the paper's results, we note that modern data of an observation cosmology do not contradict the supposition about a weak initial violation of the Universe's thermal equilibrium in terms of the condition (15), i.e., to the initial smallness of superthermal particles' number in comparison with the number of thermal ones (see the condition (15). An evolution of the non-equilibrium Universe in the case of the initial distribution's weak deviation from the thermal equilibrium, is managed generally by two dimensionless parameters: a relative contribution to the equilibrium component's energy density $0 \leqslant \sigma_0 \leqslant 1$ and a ratio of the superthermal particles' average energy to the temperature of the equilibrium Universe $1 < \overline{\mathcal{P}}_0 < +\infty$ in the same point of time. One could say also in such a way: the parameter $\overline{\mathcal{P}}_0$ is equal to the non-equilibrium particles' average energy in Planck scales of energy on Planck point of time. An observation data at that, generally the prevalence of the cosmological He_4, imposes certain restrictions on the admitted range of weakly-equilibrium model's parameters, but does not contradict values of the parameter $\overline{\mathcal{P}}_0 \lesssim 3 \cdot 10^{17}$, i.e. excesses of an average energy of superthermal particles' component in 17(!) orders of equilibrium components' temperature. Corresponding to this parameter energy in the modern Universe at a temperature of the relic radiation $3^o K$ is amount to $0,8 * 10^5$GeV(!). Our next paper will be devoted to the investigation of the Universe model with the initial strongly violated thermal equilibrium.

In conclusion authors express their thanks to professors V.N. Melnikov, Yu.S.

Vladimirov and D.V. Galtsov for a productive discussion of paper's matters.

References

[1] Yu.G.Ignatyev, *Gravitation & Cosmology* (to be publis).

[2] Yu.G. Ignat'ev, *in: "Actual theoretical and experimantal problems of relativity theory and gravitation", Report of Soviet conference*, Moscow, 1984 (in Russian).

[3] Yu.G. Ignat'ev, *Izvestiya Vuzov, Fizika* **29**, No 2 19 (1986) (in Russian).

[4] Yu.G. Ignat'ev, *Izvestiya Vuzov, Fizika* **23** No 8, 42 (1980).

[5] Yu.G. Ignat'ev, *Izvestiya Vuzov, Fizika* **23** No 9, 27 (1980).

[6] Yu.G.Ignat'ev. *Izvestiya Vuzov, Fizika*, 26, No.8, 19 (1983) (in Russian)

[7] Yu.G.Ignatyev, *in:* " Problems of Gravitation Theory and Elementary Particles", Moskow, Atomizdat, No 11 113 (1980).

[8] Yu.G.Ignat'ev. *in:* " Problems of Gravitation Theory, Relativistic Kinetics and Evolution of the Universe", KSPI, Kazan, 1988, (in Russian)

[9] H.M.Pilkuhn. *Relativistic Particle Physics*, Springer-Verlag, New York Inc, 1979

[10] Yu.G.Ignatyev, R.A.Ziatdinov. *Gravitation & Cosmology* (to be publis).

[11] N.N.Lebedev. *Special functions and its applications*, oscow, GIFML, 1963 (in Russian)

[12] Ja.B.Zeldovich, I.D.Novikov, "Structure and Evolution of Universe", Nauka, Moskow, 1975 (in Russian).

[13] A.I.Akhiezer, V.B.Berestetsky, "Quantum Electrodynamics", Moskow, Nauka, 1969 (in Russian).